Design of Experiments in Simulation Based Acquisition

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Design of Experiments (DOE)

Basic Question: How does changing Design Factor X impact MOE Y?

Identify Study Goals

- Input Factors
- MOE's

Choose Run Matrix

Simulation

- Thunder
- Brawler
- Eadsim
- JIMM
- Suppressor

DOE - a way of obtaining the maximum amount of information for the least amount of data (saves money, time, and resources)

Major Programs That Have Used DOE

JSF

- 12 input factors, each at two levels representing “best” and “worst” case
- total number of cases for all possible combinations:

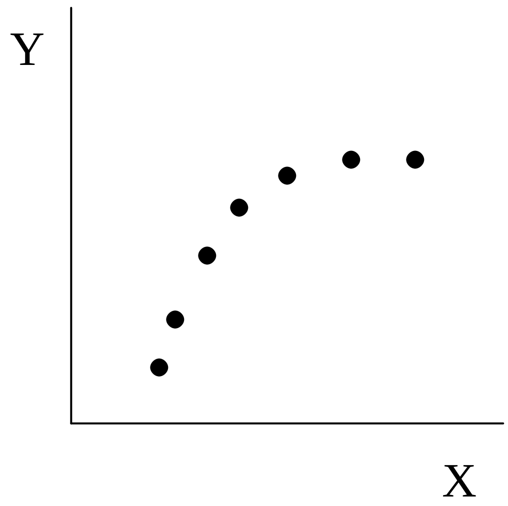
  $2^{12} = 4096$

- total number of cases with DOE: 32

CVX

JDAM

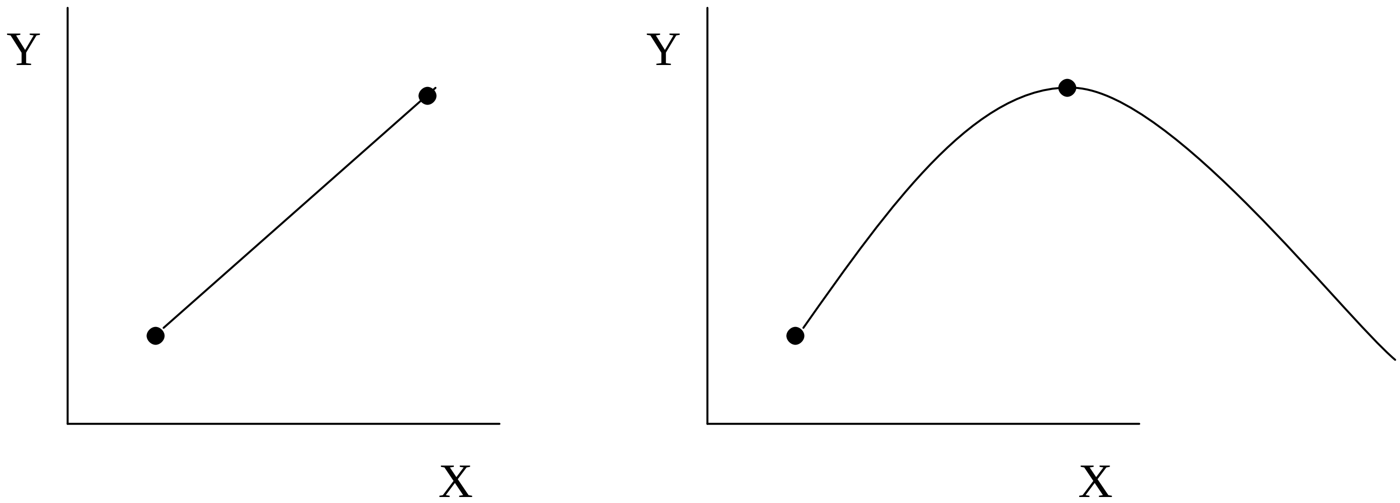
BASIC QUESTION: What is the effect on Y of changing X?



Simplest case: one factor, pick various levels of X (hold other X's constant)

Which levels of X should we pick?

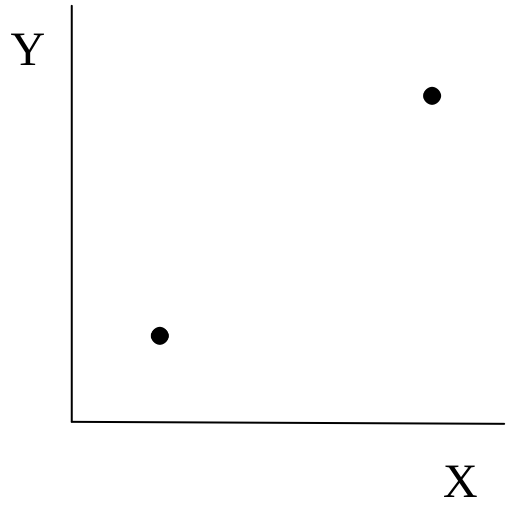
What is range of interest?



As X increases, Y increases   maybe, maybe not!

Statistical model valid only inside experimental range--cannot extrapolate!

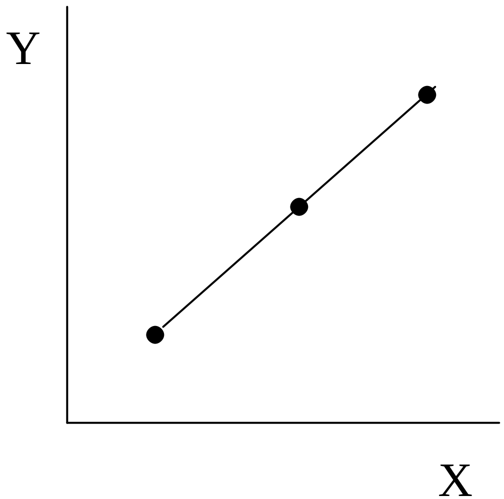
How many levels of X to pick?



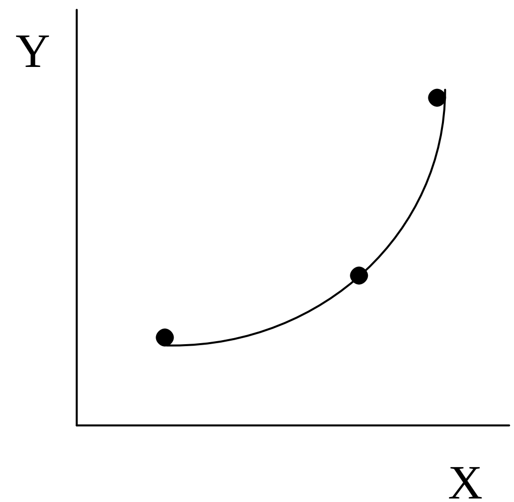
What is happening between the two points?

How many levels of X to pick?

Possibilities include:



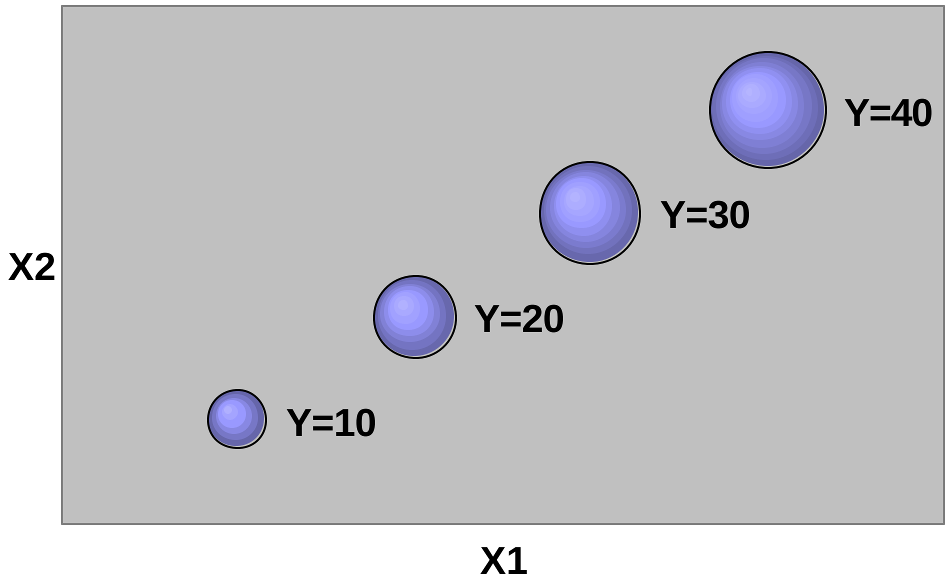
straight line



third data point indicates curvature

Definition: Confounding

What if we have more than one independent variable? Which levels should we pick then?

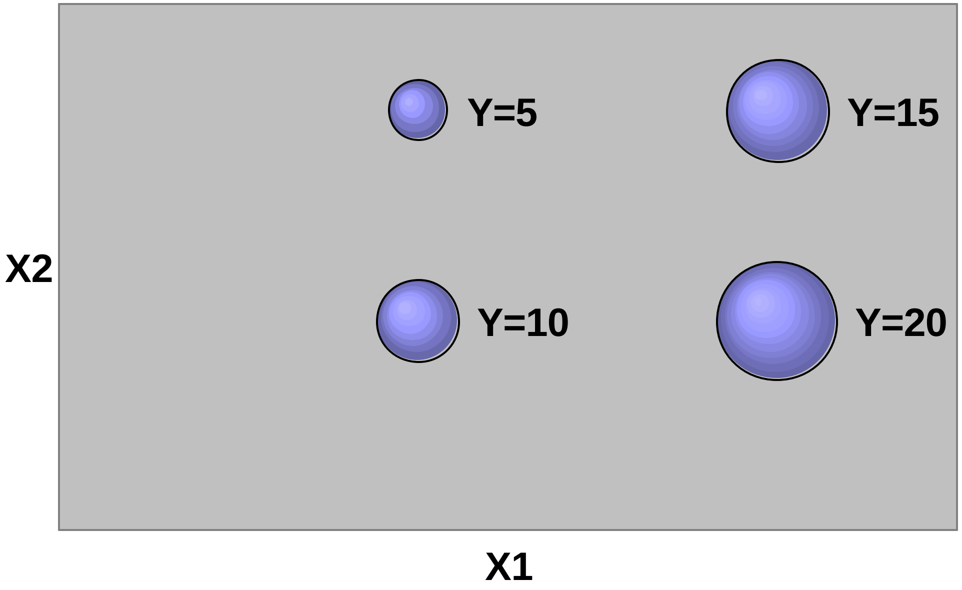


Clearly there is a trend. But is the increase in Y due to an increase in X1 or an increase in X2? Or both?

Note: In this situation, X1 and X2 are said to be **confounded**.

Definition: Orthogonal Design

A better way to set up the experiment is to use an **orthogonal** design:



Clearly there is a trend. Increasing X1 causes Y to increase, while increasing X2 causes Y to decrease.

What if you have many X's?

Example: What if you had the following design:

X1 - 7 levels
X2 - 5 levels
X3 - 2 levels
X4 - 2 levels

This design would require 7 x 5 x 2 x 2 = 140 cases!

With more variables, this can soon get out of hand.

Fortunately, **DOE offers a better way**.

Overview of DOE Process

Early stage of DOE:

Which factors are important and what is their overall effect?

Later stage of DOE:

What is the **exact** relationship between each factor and its response?

How can we optimize combinations of factor levels to maximize or minimize a response?

DOE Approach #1:

All Possible Combinations

Measure effect of one particular factor by fixing levels of remaining factors and running experiment at various levels of factor of interest.

Repeat entire process for each of the other factors, one at a time.

This allows us to measure the "**simple effect**" of X's on Y.

Problem: too many runs needed

DOE Approach #2:
2^k Factorial Design

Choose **two** levels for each of k possible factors and run experiment at each of the 2^k factor-level combinations.

This allows us to estimate the "**main effect**" of X's on Y.

Advantage: less runs required.

2^k Factorial Design

Two levels of factor are denoted by "-" or "low" level and "+" or "high" level, respectively

Example: 3 factors (2^3 design)

Run No.	X1	X2	X3	Response
1	-	-	-	R1
2	+	-	-	R2
3	-	+	-	R3
4	+	+	-	R4
5	-	-	+	R5
6	+	-	+	R6
7	-	+	+	R7
8	+	+	+	R8

Note: R1…R8 are values of the response associated with the i'th combination of factor levels

Definition: Main Effect (ME)

Formal definition: average change in the response due to moving a factor from its "-" level to its "+" level while holding all other factors fixed

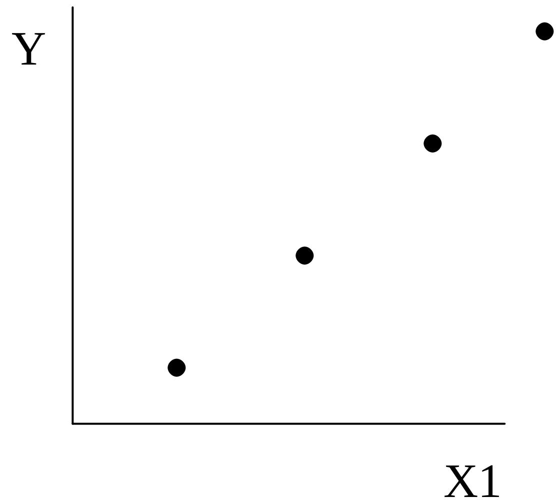
For the previous example, ME of X1 can be calculated as follows:

$$\mathbf{ME(X1)} = \frac{\mathbf{(R2 - R1) + (R4 - R3) + (R6 - R5) + (R8 - R7)}}{\mathbf{4}}$$

Definition: Interaction Effect

Sometimes two factors can interact with each other.

Consider the following case:

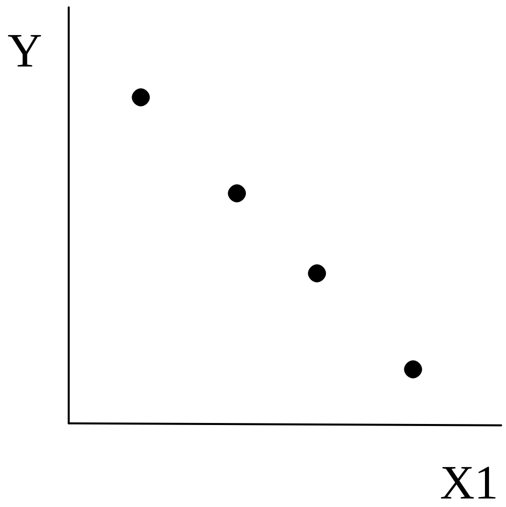


(X2 is held constant)

X1 is varied over 4 levels, X2 is held constant--Y appears to be increasing as a function of X1

Definition: Interaction Effect

Suppose we repeat the experiment for a different level of X2:



(X2 is held constant at new level)

Y appears to be decreasing as a function of X1 (the exact opposite of what we had before)

Definition: Interaction Effect

Interactions

- effect of one factor (X1) depends on level of another factor (X2)
- synergistic or antagonistic
- determined via plots or statistical tests
- higher order interactions are possible but rare

Fractional Factorial Designs

What if you have many factors?

Number of Factors (k)	Number of Cases (2^k)
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024
11	2048
12	4096

Clearly, this can get out of hand!!!

Fractional Factorial Designs

Is it really necessary to run every single combination of every single factor at every single level?

Fortunately, the answer is NO!

Fractional Factorial Designs allow us to get good estimates of main effects and interactions at a fraction of the price! (or time and effort…)

Fractional Factorial Designs

A certain subset of the 2^k possible design points are selected.

But, which ones to choose?

Theoretical statistics gives us the answer.

Computer programs do the work!

Fractional Factorial Designs

How does it work?

Price of fractional factorial: certain effects confounded with each other

Example:

Main effect confounded with interaction:
formula = ME(X4) $\pm$ X1X2X3

Assumption:

Higher order interactions are negligible w.r.t. main effects and lower order interactions

Fractional Factorial Designs

If we can assume that higher-order interactions are negligible, we don’t need to run every single case.

Subject matter expert/analyst is consulted to determine which interactions are likely to be negligible.

Design (run matrix) is chosen accordingly.

DOE Strategy

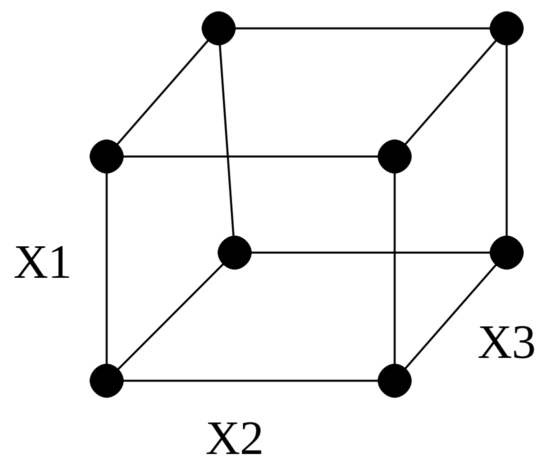
Objective: screen out unimportant factors, identify significant ones

Another name for a fractional factorial design is "**screening design**"

Original design matrix allows us to identify significant factors--BUT...

DOE Strategy

What is the exact nature of the relationship between these factors and the response?



Original design has 8 design points

DOE Strategy

Additional data points can be added to the original design.

A “follow-up” study can be done to optimize or determine sensitivity of a response.

Other types of designs exist.

Response Surface Methodology

RSM is a collection of mathematical and statistical techniques used to optimize response.

Contour/surface plots can be used to characterize the response surface.

Regression analysis is frequently used to come up with an exact equation.

Other Designs

Mixture/Simplex Designs

- levels of X's add up to 1 (not independent)
- commonly used in chemical industry
- could be used to optimize shipfill

Other Designs

Computer-Generated (Optimal) Designs

• irregular experimental design (e.g., region of interest is not a cube or a sphere due to constraints on X's)

• nonstandard model

• unusual sample size requirements

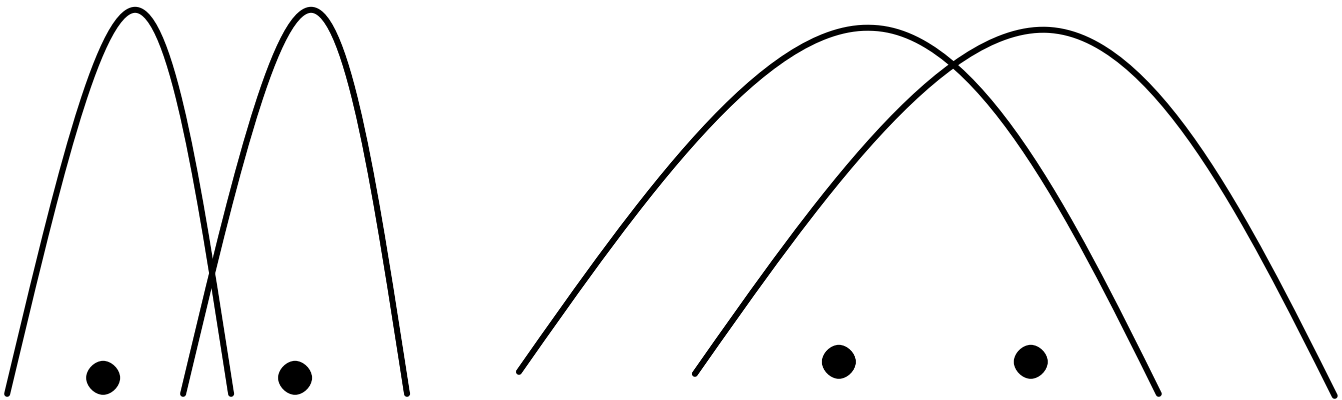
Monte-Carlo Replication

Some models use random numbers--results change with every replication

Exactly how many replicates do we need in order to distinguish the “signal” from the “noise”?

Considerations

What is the size of the random error or variability?



Difference is large w.r.t. variability--statistically significant

Difference is small w.r.t. variability--not statistically significant

Need to consider:

- difference
- random error
- statistical confidence

Note: Difference is the same in both cases! It’s all relative!

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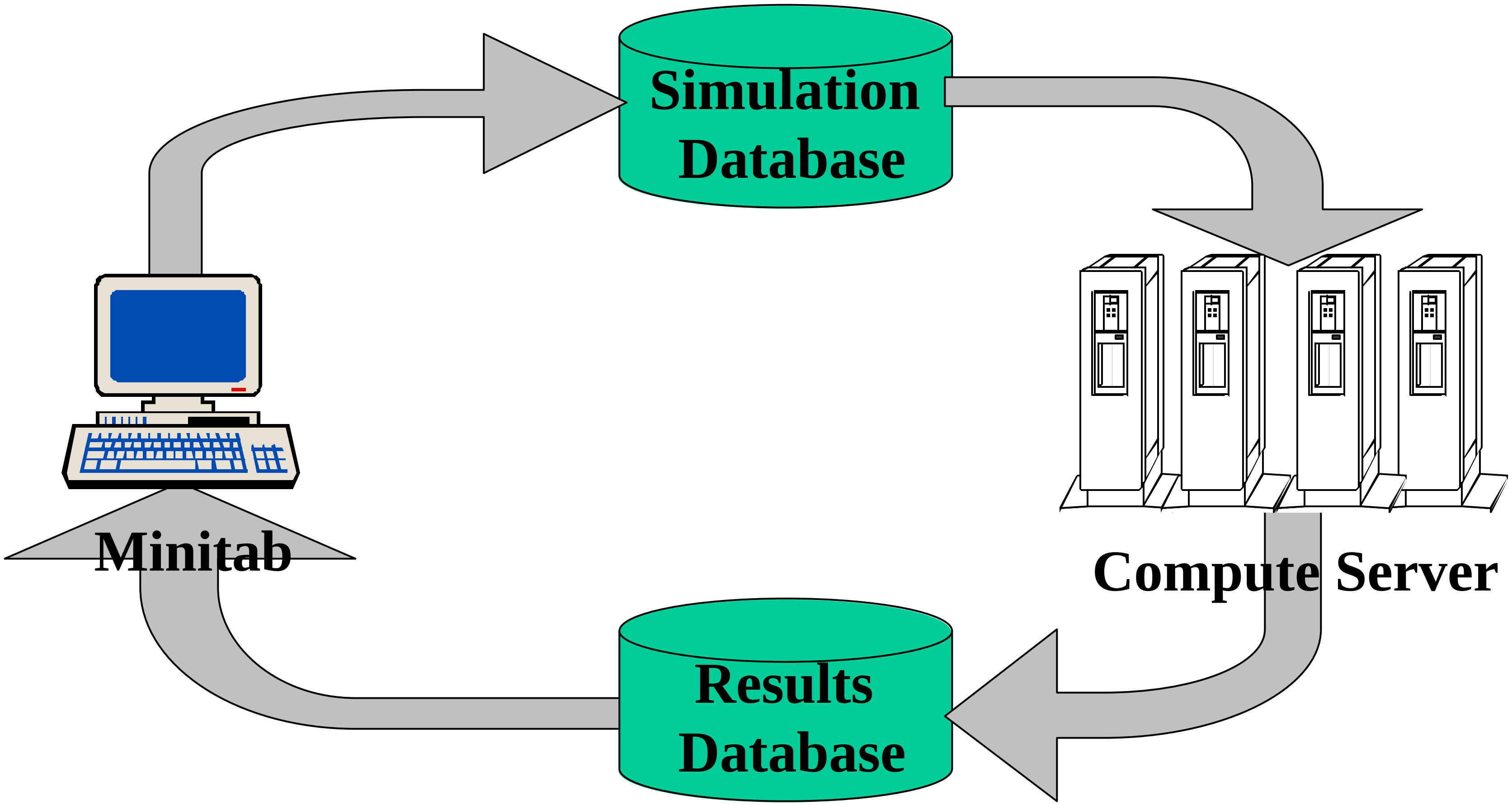
Actual Process (Typical Study)



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